Curriculum Vitae
Mauricio Velasco

Basic info:

- Name: Mauricio Fernando Velasco Gregory.
- Date of Birth: July 28, 1979 (age: 44) ; Bogotá, Colombia.
- e-mail: `mauricio.velasco@ucu.edu.uy`.
- web page: `http://wwwprof.uniandes.edu.co/~mvelasco/Velasco.html`.

Employment:

- August 2023 to present: Profesor Investigador, Universidad Católica del Uruguay, Montevideo, Uruguay.
- January 2017 to present: Associate Professor, Universidad de los Andes, Bogotá, Colombia.
- January 2016 to December 2016: Profesor Agregado, Universidad de la República (CURE), Maldonado, Uruguay.
- July 2013 to December 2016: Associate Professor, Universidad de los Andes, Bogotá, Colombia.
- June 2011 to July 2013: Assistant Professor, Universidad de los Andes, Bogotá, Colombia.
- February 2010 to June 2011: Associate, Goldman-Sachs International, Derivatives Analysis Group, London, UK.
- July 2007 to January 2010: Morrey Assistant Professor, UC Berkeley, Department of Mathematics, Berkeley, CA, USA.

Education:

- Cornell University:
  PhD Mathematics (2007)
  PhD Supervisor: Michael Stillman
  M.S. Mathematics, 2004
- Universidad de los Andes (Bogota): B.Sc., Mathematics (Cum Laude) (2002)
  Minor in economics, (2002)

Research Area: Applied Algebraic Geometry. My main areas of interest are the relationship between Classical Algebraic Geometry and other fields: initially combinatorics and symbolic computation and lately optimization, coding theory and machine learning.

Scientific Leadership and Awards:

- Invited lecture series:
  - *"Polynomial optimization on finite sets"*, Lecture series for the thematic semester "Current themes of discrete optimization" at ICERM, January 2023, (Brown University, Providence, USA)
  - *"Sums of squares: theory and applications"*, Lecture series organized by the American Mathematical Society for the USA Joint meetings, January 2019, (Baltimore, USA).
- Recent plenary talk invitations:
  - *Iterative games on graphs and the protection of National parks*, CICADA Artificial Intelligence Center, Universidad de la República, November 2022 (Punta del Este, Uruguay).
  - *The geometry of sums-of-squares,* Cornell University, Applied Mathematics Colloquium, May 2021 (Cornell University, USA).
  - *Nonnegative polynomials on algebraic curves and surfaces,* Mathematical Congress of the Americas (MCA 2021), July 2021 (Buenos Aires, Argentina).
  - *Some Vignettes of sums-of-squares on varieties,* Western Algebraic Geometry symposium Online (WAGON), April 2020 (University of Washington, USA).

- *Extremal properties of* 2*-regular varieties,* SIAM conference on applied algebraic geometry, October 2019 (Bern, Switzerland)

- Awards:
  - "José Fernando Escobar" prize for research in Mathematics 2017 (Awarded every two years by the Sociedad Colombiana de Matemáticas for the most significant national achievements in pure and applied mathematics).
  - ACCEFIN-TWAS Prize for young scientists in developing countries (Colombia) 2016. Awarded by the Academia Colombiana de Ciencias Exactas, Físicas y Naturales for exceptional research in the natural sciences and engineering.
  - Distinguished Undergraduate Teaching Award, Department of mathematics, 2008 (UC Berkeley, USA).
  - Hutchinson Award, Department of mathematics, 2006 , (Cornell University, USA).

Research Articles: (*In mathematics author order is always alphabetical*)**:**

1. On random walks and switched random walks in homogeneous spaces (with E. Moreno) *To Appear in Combinatorics, Probability and Computing,(2022).*
2. Smart Pooling: AI-powered COVID-19 testing (with J. Madrid-Wolff, P. Arbelaez, M. Forero-Shelton, J.M. Pedraza, et al.) *Nature Scientific Reports, Vol. 12 (2022).*
3. Constructing Partial MDS Codes from Reducible Curves (with T.Bogart, A.L. Horlemann-Trautmann, D. Karpuk and A. Neri) *SIAM Journal on Discrete Mathematics, Vol. 35, Issue 4, pp. 2946-2970, (2021).*
4. Sums of Squares: A Real Projective Story (with G.Blekherman, R.Sinn and G.G.Smith) *Notices of the AMS, Vol. 68, Issue 5, pp. 734-747, (2021).*
5. Sums of Squares and Quadratic Persistence on Real Projective Varieties (with G.Blekherman, R.Sinn and G.G.Smith) *Journal of the European Mathematical Society Vol. 24, Issue 3, pp. 925-965, (2021).*
6. Approximate super-resolution of positive measures in all dimensions (with C. Hernandez, H. Garcia and M. Junca) *Applied and Computational Harmonic Analysis, Vol. 52, pp. 251-278, (2020).*
7. Algebraic Geometry and sums of squares, *Proceedings of Symposia in Applied Mathematics (PSAPM), part of the volume entitled Sums of Squares: Theory and Applications edited by P. Parrilo and R. Thomas.* (2020).
8. Local angles and dimension estimation from data on manifolds (with M. Diaz and A.J.Quiroz), *Journal of Multivariate Analysis, 173, pp.229-247 (2019).*
9. Sharp Degree Bounds for Sum-of-Squares Certificates on Projective Curves (with G. Blekherman and G.G. Smith), *Journal des Mathematiques Pures et Appliquees (JMPA), Vol. 129, pp.61-86, (2019).*
10. Compressed sensing of data with a known distribution (with M. Diaz, M. Junca and F. Rincón), *Applied and Computational Harmonic Analysis, Vol. 45, Issue 3, pp. 486-504 (2017).*
11. Do sums-of-squares dream of free resolutions? (with G. Blekherman and R. Sinn), *SIAM Journal on Applied Algebra and Geometry (SIAGA), Vol. 1, Issue 1, pp. 175-199 (2017).*
12. Semidefinite approximations of conical hulls of measured sets (with J.Romero), *Discrete and Computational Geometry, Vol. 57, pp.71-103 (2017)*
13. A lower bound for the determinantal complexity of a hypersurface (with J. Alper and T. Bogart), *Foundations of Computational Mathematics (FOCM), Vol.17, pp. 829-836 (2017)*
14. Test sets for nonnegativity of polynomials invariant under a finite reflection group (with J.G. Acevedo), *Journal of Pure and Applied Algebra, Vol. 220, Issue 8, pp. 2936-2947 (2016).*
15. Sums of squares and varieties of minimal degree (with G. Blekherman and G.G. Smith), *Journal of the American Mathematical Society (JAMS), Vol. 29 , pp.893-913 (2016).*
16. Gap vectors of real projective varieties (with G.Blekeherman, S. Iliman and M. Kubitzke), *Advances in Mathematics, Vol. 283, Issue 1, pp. 458-472 (2015).*
17. Dual toric codes and polytopes of degree one (with V. Gauthier), *SIAM Journal on Discrete Mathematics*, Vol. 29, Issue 1, pp. 683-692, (2015).
18. Linearization functors in real convex sets, *SIAM Journal on Optimization*, Vol.25, Issue 1, pp. 1-27, (2015)

19. Estimation of seed shadows generated by Andean woolly monkeys (with Onshuus A., Link A., Stevenson P., Quiroz A.) *International Journal of Primatology, Vol. 35, Issue 5, pp. 1021-1036 (2014).*
20. The maximum cut problem for blow-ups of multiprojective space (with M. Junca), *Journal of Algebraic Combinatorics, Vol. 38, Issue 4, pp. 797-827 (2013).*
21. Big rational surfaces (with D. Testa and T. Varilly), *Math. Annalen, Vol. 351, Issue 1, pp. 95-107, (2011).*
22. Frames and degenerations of monomial ideals (with I. Peeva), *Trans. Amer. Math. Soc. Vol. 363, Issue 4, pp. 2029-2046 (2011).*
23. Blow-ups of $\mathbb{P}^{n-3}$ at n points and Spinor varieties (with B. Sturmfels), *Journal of Commutative Algebra Vol.2, pp. 223-244, (2010).*
24. A syzygetic approach to the smoothability of zero-dimensional schemes (with D. Erman), *Advances in Mathematics , Vol. 224, Issue 3, pp. 1143-1166, (2010).*
25. The Cox rings of degree one Del Pezzo surfaces (with D. Testa and T. Varilly), *Algebra and Number Theory, Vol. 3, Issue 7, pp. 729-761, (2009).*
26. Hilbert schemes of 8 points (with D. Cartwright, D. Erman and B. Viray), *Algebra and Number Theory, Vol. 3, Issue 7, pp. 763-795, (2009).*
27. Picard graded Betti numbers and the defining ideals of Cox rings, (with A. Laface), *Journal of Algebra, Vol.322, Issue 2, pp. 353-372, (2009).*
28. A Survey on Cox rings (with A. Laface) *Geometriae Dedicata, Vol. 139, Issue 1, pp. 269-287 (2009).*
29. Minimal free resolutions that are not supported by a CW complex. *Journal of Algebra, Vol. 319, Issue 1 pp. 102-114, (2008).*
30. Gröbner bases, monomial group actions and the Cox rings of Del Pezzo surfaces (with M. Stillman and D. Testa), *Journal of Algebra, Vol. 316, Issue 2, pp. 777-801, (2007).*

Submitted Articles:

1. Harmonic hierarchies for polynomial optimization (with S. Cristancho)
`https://arxiv.org/abs/2202.12865`
2. Minimum cross-entropy distributions in Wasserstein balls and their applications (with L.F. Vargas)
`https://arxiv.org/abs/2106.03226`
3. Moment-SOS hierarchy and the exit time of Stochastic processes (with D. Henrion and M. Junca)
`https://arxiv.org/abs/2101.06009`

Grants:

- ECOSNord Colombia - France cooperation grant (2019 - December 2022).
- Proyecto FAPA Universidad de los Andes: From (2011-2) to present.
- National Science Foundation (NSF, USA) full grant 2008-2010: Program in Algebra, Number Theory and Combinatorics, grant DMS-0802851 "Combinatorial Commutative Algebra of Cox Rings and the Hilbert Scheme of Points" (with D. Eisenbud).
- Cornell Graduate Fellowship 2002-2003.

Invited talks during the last five years (in reverse chronological order):

- *Harmonic hierarchies for polynomial optimization,* Polynomial Optimization Seminar (BrainPOP), LAAS-CNRS Toulouse, November 2021 (Virtual, Toulouse, France)
- *Harmonic hierarchies for polynomial optimization,* Optimization Seminar, Tepper Business School, November 2021, (Virtual, Carnegie Mellon University, USA)
- *Nonnegative polynomials on algebraic curves and surfaces,* Mathematical Congress of the Americas (MCA 2021), July 2021 (Virtual, Buenos Aires, Argentina).
- *The geometry of sums-of-squares,* Cornell University, Applied Mathematics Colloquium, student selected speaker, May 2021 (Virtual, Cornell U., USA).
- *The geometry of sum-of-squares on varieties,* Nonlinear Algebra Seminar Online, February 2021 (Virtual, Max Planck Institute, Leipzig, Germany).

- *Optimal pool testing protocols and online learning,* Seminario de probabilidad y estadística de Montevideo, July 2020 (Virtual, UDELAR, Uruguay).
- *Optimal pool testing protocols for COVID-19,* FAPESP Seminars for COVID-19, Focusing on maths for COVID-19 on Southamerica, June 2020 (Virtual, FAPESP, Brazil).
- *Some Vignettes of sums-of-squares on varieties,* Western Algebraic Geometry symposium Online (WAGON), April 2020 (Virtual, U. Washington, USA).
- *On maximum entropy distributions in Wasserstein balls,* poster presented at KHIPU Latin American Conference In Artificial Intelligence, November 2019 (Montevideo, Uruguay)
- *Extremal properties of* 2*-regular varieties,* Plenary talk at the bi-annual meeting SIAM conference on applied algebraic geometry, October 2019 (Bern, Switzerland)
- *Distribuciones de máxima entropía en Bolas de Wasserstein,* Congreso Colombiano de Matemáticas, Junio 2019 (Popayán, Colombia).
- *Lecturer for the course: "Sums of squares: theory and applications"*, January 2019, (Baltimore, Maryland, USA) organized by the American Mathematical Society.
- *Pythagoras numbers of projective varieties,* Real algebraic geometry and optimization workshop (ICERM), October 2018 (Providence, Rhode Island, USA)
- *Aprendizaje de máquina sobre grafos y optimización robusta,* MAPI1 Conferencia Colombiana de Matemáticas Aplicadas e Industriales, August 2018 (Bogotá, Colombia)
- *Pythagoras numbers of projective varieties,* Plenary talk at ECCO Encuentro Colombiano de Combinatoria, June 2018 (Barranquilla, Colombia)

Selected invited talks (prior to 2017):

- *Some monomial ideals associated to simplicial complexes*, Route 81 conference, Queens University, Fall 2005
- *Some monomial ideals associated to simplicial complexes*, Western sectional of the AMS meeting, Eugene, Fall 2005
- *Does every minimal free resolution admit a cellular structure?*, Commutative Algebra Seminar, University of Nebraska at Lincoln, Spring 2006
- *Nearly Scarf ideals and CW resolutions*, Summer school on monomial ideals and Hilbert functions, Cornell University, Summer 2006
- *The homogeneous coordinate rings of some Del Pezzo surfaces*, Conference in combinatorial and computational commutative algebra, Fields Institute, Toronto, summer 2006
- *The homogeneous coordinate rings of some Del Pezzo surfaces*, Route 81 conference, Syracuse University, Fall 2006
- *The homogeneous coordinate rings of some Del Pezzo surfaces*, Banff International Research Station workshop on Syzygies and Hilbert functions, Fall 2006
- *Gröbner basis, monomial group actions and the Cox rings of Del Pezzo surfaces*, Queen's University Algebraic Geometry seminar, Fall 2006
- *Gröbner basis, monomial group actions and the Cox rings of Del Pezzo surfaces*, Valley geometry seminar, University of Massachusetts at Amherst, Fall 2006
- *Gröbner basis, monomial group actions and the Cox rings of Del Pezzo surfaces*, Special Session on Commutative Algebra and Algebraic Geometry at the Canadian Mathematical Society Winter Meeting, Toronto, Fall 2006
- *The homogeneous coordinate rings of some Del Pezzo surfaces*, Joint meetings of the AMS Special Session on Commutative Algebra and Algebraic Geometry, New Orleans, Spring 2007
- *Some monomial ideals associated to simplicial complexes*, Bay Area Discrete Math Day, BADMath, Google campus, Fall 2007
- *The Cox rings of degree one Del Pezzo surfaces*, Stanford Algebraic Geometry Seminar, Spring 2008
- *Big rational surfaces and multigraded Hilbert functions of points on toric surfaces*, Number Theory Seminar, University of Arizona, Spring 2008
- SIAM annual meeting 2012 (Minneapolis):
  - *Linearization functors on real convex sets,* special session in Convex Algebraic Geometry.

  - *On the maximum cut problem for blow-ups of multiprojective space,* special session in Combinatorial Algebraic Geometry.
- *Spinor varieties and blow-ups of $\mathbb{P}^n$ at $n+3$-points,* IV CLAM Congreso Latinoamericano de Matemática 2012 (Córdoba, Argentina).
- *On the maximum cut problem for blow-ups of multiprojective space,* ECCO, Encuentro colombiano de Combinatoria 2012 (Bogotá, Colombia).
- *Introducción a la optimización semidefinida,* Charla inaugural del Encuentro colombiano de optimización, Universidad Sergio Arboleda 2012 (Bogotá, Colombia).
- *Nonnegative quadrics and sums of squares on real projective varieties,* Coloquios de Geometria e Aritmetica 2013 (IMPA, Brasil).
- *When is every nonnegative section a sum of squares?,* SIAG bi-annual meeting 2013 (Colorado, USA).
- *Nonnegative sections and sums of squares on real projective varieties,* 2014 NIMS Thematic program on applied algebraic geometry plenary talk, (Daejeon, Korea).
- At FOCM 2014: Foundations of Computational Mathematics conference (Montevideo, Uruguay),
  - *Nonnegative forms on real algebraic curves and global optimization* (Continuous optimization session).
  - *Dual toric codes* (session on Computational Algebraic Geometry).
- *On Hilbert's 17th problem on algebraic curves,* II Latin American School of Algebraic Geometry and applications, 2015 (Rio de Janeiro, Brasil).
- *Polynomial optimization on algebraic curves*, ISMP 2015: 22nd International Symposium on Mathematical Programming (Pittsburgh, USA).
- *Do sums of squares dream of free Resolutions?*, Congreso Latinoamericano de Álgebra, 2016 (Buenos Aires, Argentina).
- *Weighted compressive sensing*, 2016, CABIDA Big DATA, (Montevideo, Uruguay).
- *Do sums of squares dream of free Resolutions?*, SIAM meeting on Optimization, 2017 (Vancouver, Canada).
- *Do sums of squares dream of free Resolutions?*, SIAM Algebraic Geometry bi-anual meeting 2017 (Atlanta, Georgia)

Students and Postdocs supervised: Mentoring is one of the activities I enjoy the most. I have had the privilege of carrying it out at all levels. Below you can see my students' thesis titles and their first placement after completing their mentoring under my supervision.

- Postdocs Supervised:
  - (2014-2015) Felipe Rincón (Combinatorics, Tropical geometry) currently a professor at the School of Mathematical Sciences at Queen Mary U. (London, UK).
  - (2013) Valérie Gauthier (Coding Theory). Currently a professor of Engineering (informatics) at Universidad de los Andes (Bogotá, Colombia)
- Phd students:
  - Javier Hernán García (ongoing, expected graduation date 2024-2)
  - Nicolás Castro (ongoing, expected graduation date 2025-2)
- Masters' students:
  - (2012) Juan Sebastian Osorio, *Teoria de Morse discreta y equivariante*; Went to become a PhD student at Universidad de los Andes (Colombia)
  - (2013) Julian Romero, *A semidefinite approximation for the traveling salesman polytope.* Went to pursue a PhD in the Combinatorics and Optimization program at the University of Waterloo (Canada)
  - (2014) Jose Dario Bastidas, *On the Cox rings of Big rational surfaces,* went to pursue a PhD in Pure Mathematics at Cornell University (USA)
  - (2015) Sergio Camelo, *Semidefinite approximations of copositive programs.* Went to pursue a PhD in Applied Mathematics at Stanford University (USA)

- (2015) Jose Gabriel Acevedo, *Test sets for nonnegativity of polynomials invariant under a finite reflection group.* Went to pursue a PhD in Pure Mathematics at the Georgia Institute of Technology (USA)
- (2016) Mateo Diaz, *Weighted compressive sensing.* Went to pursue a PhD in Applied Mathematics at Cornell University (USA)
- (2016) Javier Sánchez, *Deterministic constructions of compressive sensing matrices.* went to pursue a PhD at Universidad de los Andes (Colombia)
- (2016) Jorge Olarte, *Rational Harnack curves.* Went to pursue a PhD in Pure Mathematics at the Berlin Mathematical School (Germany)
- (2018) Daniel De Roux, *Graph Learning and the Nuclear Wasserstein Metric,* Went to pursue a PhD in the ACO program at Carnegie Mellon (USA)
- (2019) Luis Felipe Vargas Beltrán, *Maximum entropy distributions on Wasserstein balls*, went to pursue a PhD in pure mathematics in the POEMA inter-european training network.
- (2020) Nicolás Castro, *Hermitian sums-of-squares in roots of unity hypercubes*, went to pursue a PhD at Universidad de los Andes.
- (2022) Nicolás Betancourt, *Green Security games along trails*, currently applying for admission to PhD programs.
- (2022) Diego Arévalo, *Rank minimization and Pythagoras' numbers*, went to work as a senior data scientist at Melius I+D.
- (2022) Sergio Cristancho, *Harmonic hierachies for polynomial optimization*, admitted as a future PhD student at Princeton University.

- Undergraduate Theses supervised:
  - (2012) José Gabriel Acevedo, *Representaciones polinomiales de* $\mathrm{GL}(n, \mathbb{C})$; Fabián Latorre, *A generalization of the maximum flow problem.*; Jose Dario Bastidas *Curvas algebráicas: Resolución de singularidades y clasificación.*
  - (2014) Javier Sánchez, *Algorithmic computation of splitting fields and Galois groups.*
  - (2015) Mónica Ribero, *Monte Carlo methods for hypothesis testing on contingency tables.* Went to pursue a PhD in Electrical Engineering at UT Austin (USA).
  - (2018) Elvira Moreno, *Equivariant polynomial norms and semidefinite programming.* Went to pursue a PhD in Applied Mathematics at Caltech (USA).
  - (2022) Verónica Calvo, *The Vershik-Okounkov approach to representation theory.* Recently admitted as a PhD student at Cambrige University.

Scientific events organized (in reverse chronological order):

- Currently I am a *program committee member* for the SIAM Applied Algebraic geometry conference (this is the bi-annual meeting of the group). The conference will take place on July 2023 at TU Eindhoven (the Netherlands).
- I am one of the creators and main organizers of the *MAPI: Conferencia colombiana de matemáticas aplicadas e industriales* conference. It is a bi-annual national conference intended to include all areas of pure and applied mathematics.
  1. MAPI 2: Medellín (June 2022).
  2. MAPI 1: Bogotá (2018).
- Member of the *MEGA 2019 (Effective Methods in Algebraic Geometry) Executive Committee.*
- Organizer of *Computational Algebra and Applications of Algebra.* Special session in the XXII Congreso Latinoaméricano de Álgebra August 7-9, 2017, Quito, Ecuador.
- Organizer and lecturer *Summer school on the Geometry of Sums-of-Squares and its applications.* July 10-15, 2016, Georgia Institute of Technology, Atlanta, Georgia.

- *School on Cryptography*, November 15-18, 2011, Universidad de los Andes, Bogotá, Colombia.

Teaching experience: I have taught courses from a wide array of areas in pure and applied mathematics. In chronological order: At *Cornell U.*: Differential Equations, Multivariable calculus. At *UC Berkeley*: Complex Analysis, Linear Algebra 2, Abstract Algebra, Algebraic Topology, Combinatorics, Introduction to Algebraic Geometry. At *Universidad de los Andes*: Álgebra Abstracta 2, Cálculo Integral, Álgebra Conmutativa, Cálculo Vectorial, Geometría Algebraica I, Geometria Algebraica II, Optimización Lineal, Optimización Convexa, Optimización Convexa II, Variedades Tóricas I, Algoritmos en Teoría de Invariantes, Variedades Tóricas I, Teoría de representaciones, Análisis numérico, Introducción al Aprendizaje por refuerzo (reinforcement learning).

Editorial Experience:

- Since January 2021 I have been an Associate Editor for SIAGA (SIAM Journal on Applied algebra and Geometry). *Please do send us your best work in these areas.*
- I have served as a reviewer for the following wide range of journals:
  Advances in Mathematics, Algebra and Number Theory, Discrete and Computational Geometry, Transactions of the AMS, Journal of Commutative Algebra, Journal of Algebraic Combinatorics, Journal of Symbolic Computation, Bulletin of the London Mathematical Society, Linear Algebra and its Applications, Proceedings of the AMS, SIAM Journal on optimization, Foundations of Computational Mathematics, Information and Inference, SIAM Journal on Applied Algebra and Geometry.

Consulting experience: Over the last five years I have been involved in several consulting projects with the colombian start-up Melius I+D. I have served as research consultant (lead) for a team of data scientists/developers for building the following two web applications:

- NOVO: Is an application for planning and supply chain logistics for Huevos Kike's (one of Colombia's largest egg producers, with an output of 4 million eggs per day). Mathematically the project involved a mixture of statistics and optimization, requiring the development of yield estimation and of egg demand models as well as solving large-scale integer programs for the demand matching problem. Additionally the problem required the implementation of good heuristics for variations of the traveling-salesman problem.

- JAMPI: Is an AI language model for helping accounting professionals in the health-care system with the billing processes so as to automatically detect and reduce anomalies. Mathematically the project involved the development of deep learning methods (word-embeddings and variational auto-encoders) capable of learning in an unsupervised manner the statistical patterns of medical bills. Anomalies with respect to those patterns get flagged and reported to users in real-time.

Miscellaneous information:

- Born: July 28th 1979 (age: 43), Bogota, Colombia.
- Languages: fluent in Spanish (native tongue), English and Italian. Beginner competence in French.
- Programming Skills: My research and professional experience have involved the systematic use of computers as means for exploring mathematical conjectures, for the development of novel algorithms, and in the construction of software for several consulting projects. As a result,
  - I am an expert Python programmer with extensive knowledge of several specialized libraries (especially for optimization, coding theory, symbolic computation and machine learning).
  - Proficient in Julia (for high-performance computing), Macaulay2 (specialized software for symbolic computation in algebraic geometry) and LaTex (for scientific typewriting).
  - Intermediate back-end web developer (Django, Flask).
- Memberships: Society for industrial and applied mathematics (SIAM).